BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

B70 06082

SUBJECT: Analytical Development of the Skylab CMG Rotation Law - Case 620

DATE: June 29, 1970

FROM: J. Kranton

ABSTRACT

The purpose of this memorandum is to present an analytical development of the CMG rotation law given in MSFC's Interface Program Requirements Document. This law was developed by MSFC using geometrical arguments alone. The analytical development here reveals that certain simplifications in the rotation law, not apparent from the geometry, are possible.

(NASA-CR-113130) ANALYTICAL DEVELOPMENT OF THE SKYLAB CMG ROTATION LAW, CASE 620 (Bellcomm, Inc.) 8 p

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MEMORANDUM FOR FILE

INTRODUCTION

The purpose of the CMG rotation law is to generate a set of commanded gimbal angle rates that

1. produce zero control torque
2. act to minimize gimbal angles.

This memorandum assumes intimate knowledge of the Skylab CMG system.

GIMBAL ANGLE RATES FROM RELATIVE ANGULAR VELOCITY

Let*

$\underline{h}_i$ = unit vector parallel to spin axis of i^{th} CMG

It can be shown that

$$\dot{\underline{\gamma}}_i = \begin{pmatrix} \dot{\beta}_i \\ \dot{\alpha}_i \end{pmatrix} = R_i \, \underline{\omega}_i \; ; \; i=1,2,3 \qquad (1)$$

where

$\underline{\omega}_i$ = angular velocity of i^{th} CMG relative to the spacecraft

$\dot{\underline{\gamma}}_i$ = gimbal angle rates that produce the component of $\underline{\omega}_i$ perpendicular to the i^{th} CMG spin axis

*The symbols used here differ from those in the IPRD. These differences are defined in Table I and on Fig.1.

$$R_1 = \begin{pmatrix} -\frac{1}{c\beta_1}h_{12} & \frac{1}{c\beta_2}h_{11} & 0 \\ -\frac{s\beta_1}{c^2\beta_1}h_{11} & -\frac{s\beta_1}{c^2\beta_1}h_{12} & -1 \end{pmatrix} = \begin{pmatrix} -s\alpha_1 & c\alpha_1 & 0 \\ -t\beta_1 c\alpha_1 & t\beta_1 s\alpha_1 & -1 \end{pmatrix}$$

$$R_2 = \begin{pmatrix} 0 & -\frac{1}{c\beta_2}h_{23} & \frac{1}{c\beta_2}h_{22} \\ -1 & -\frac{s\beta_2}{c^2\beta_2}h_{22} & -\frac{s\beta_2}{c^2\beta_2}h_{23} \end{pmatrix} = \begin{pmatrix} 0 & -s\alpha_2 & c\alpha_2 \\ -1 & -t\beta_2 c\alpha_2 & t\beta_2 s\alpha_2 \end{pmatrix}$$

$$R_3 = \begin{pmatrix} \frac{1}{c\beta_3}h_{33} & 0 & -\frac{1}{c\beta_3}h_{31} \\ -\frac{s\beta_3}{c^2\beta_3}h_{31} & -1 & -\frac{s\beta_3}{c^2\beta_3}h_{33} \end{pmatrix} = \begin{pmatrix} c\alpha_3 & 0 & -s\alpha_3 \\ t\beta_3 s\alpha_3 & -1 & -t\beta_3 c\alpha_3 \end{pmatrix}$$

PRODUCING ZERO CONTROL TORQUE

We now wish to choose the $\underline{\omega}_i$'s so as to produce zero total torque. As Kennel does, let us rotate CMG pair A (CMGs 1 and 2) about $(\underline{h}_1+\underline{h}_2)$; i.e., let

$$\underline{\omega}_{12} = a(\underline{h}_1+\underline{h}_2)$$

Similarly, for CMG pair B let

$$\underline{\omega}_{23} = b(\underline{h}_2+\underline{h}_3)$$

and for CMG pair C let

$$\underline{\omega}_{31} = c(\underline{h}_3+\underline{h}_1)$$

Thus we may choose the $\underline{\omega}_i$'s as follows to produce zero total torque

$$\underline{\omega}_1 = \underline{\omega}_{12} + \underline{\omega}_{31} = a(\underline{h}_1+\underline{h}_2) + c(\underline{h}_3+\underline{h}_1)$$

$$\underline{\omega}_2 = \underline{\omega}_{23} + \underline{\omega}_{12} = b(\underline{h}_2+\underline{h}_3) + a(\underline{h}_1+\underline{h}_2)$$

$$\underline{\omega}_3 = \underline{\omega}_{31} + \underline{\omega}_{23} = c(\underline{h}_3+\underline{h}_1) + b(\underline{h}_2+\underline{h}_3)$$

But since

$$R_1 \underline{h}_1 = R_2 \underline{h}_2 = R_3 \underline{h}_3 = \underline{0} ,$$

we can define the $\underline{\omega}_i$'s more simply as

$$\begin{aligned} \underline{\omega}_1 &= a\ \underline{h}_2 + c\ \underline{h}_3 \\ \underline{\omega}_2 &= b\ \underline{h}_3 + a\ \underline{h}_1 \\ \underline{\omega}_3 &= c\ \underline{h}_1 + b\ \underline{h}_2 \end{aligned} \qquad (2)$$

We see that we can choose the three numbers a,b, and c arbitrarily and produce zero torque provided the $\underline{\omega}_i$'s are chosen as above. The fact that the $\underline{\omega}_i$'s given in (2) produce zero torque is easily verified by substituting into the control torque equation.

$$\underline{T}_c = \dot{\underline{H}} = H(\tilde{\underline{\omega}}_1\ \underline{h}_1 + \tilde{\underline{\omega}}_2\ \underline{h}_2 + \tilde{\underline{\omega}}_3\ \underline{h}_3) \qquad (3)$$

We can now write

$$\begin{aligned}
\dot{\underline{\gamma}}_1 &= R_1(\underline{h}_2 \,|\, \underline{h}_3)\binom{a}{c} \\
\dot{\underline{\gamma}}_2 &= R_2(\underline{h}_1 \,|\, \underline{h}_3)\binom{a}{b} \\
\dot{\underline{\gamma}}_3 &= R_3(\underline{h}_2 \,|\, \underline{h}_1)\binom{b}{c}
\end{aligned} \tag{4}$$

CHOICE OF a, b, AND c

Let us define

$$\underline{\gamma}_i^n = \begin{pmatrix} \beta_i^n \\ (\alpha_i - \frac{\pi}{4})^n \end{pmatrix} ; \; n = \text{odd integer} \tag{5}$$

where 0 and $\frac{\pi}{4}$ are inner and outer gimbal angles when $\underline{H}=\underline{0}$. Further, let us define a performance index J as

$$\begin{aligned}
J = \sum_{i=1}^{3} (\underline{\gamma}_i^n)' \dot{\underline{\gamma}}_i &= a[(\underline{\gamma}_1^n)' R_1 \underline{h}_2 + (\underline{\gamma}_2^n)' R_2 \underline{h}_1] \\
&+ b[(\underline{\gamma}_2^n)' R_2 \underline{h}_3 + (\underline{\gamma}_3^n)' R_3 \underline{h}_2] \\
&+ c[(\underline{\gamma}_1^n)' R_1 \underline{h}_3 + (\underline{\gamma}_3^n)' R_3 \underline{h}_1] \\
&\equiv a\lambda_a + b\lambda_b + c\lambda_c
\end{aligned} \tag{6}$$

Since we wish to minimize J, the signs of a, b, and c should be chosen according to

$$\operatorname{sgn} j = -\operatorname{sgn} \lambda_j \; ; \qquad j = a,b,c \tag{7}$$

Clearly, one sensible choice for a, b, and c is

$$j = -K_R \lambda_j ; \qquad j = a,b,c \tag{8}$$

which is precisely the choice reflected in equations 9.3.3. 62,63,64 in the IPRD. Another sensible choice is

$$j = -K_R(\lambda_j)_{max} \operatorname{sgn} \lambda_j ; \qquad j = a,b,c \tag{9}$$

where $(\lambda_j)_{max}$ is the maximum allowed value of λ_j. A good compromise between (8) and (9) is

$$\begin{aligned} |\lambda_j| &\geq (\lambda_j)_{max}, \quad \text{use (9)} \\ |\lambda_j| &< (\lambda_j)_{max}, \quad \text{use (8)} \end{aligned} \tag{10}$$

Kennel's control law is more involved than (10). He applies bounds to the components of the $R_i \underline{h}_j$ vectors in (6); i.e., to the S's shown on pages 14-16 in the IPRD. Of what value Kennel's complication is, is not clear.

Kennel also specifies a rotation about the total angular momentum vector (eq. 9.3.3.65); i.e.,

$$\underline{\omega}_T = d(\underline{h}_1 + \underline{h}_2 + \underline{h}_3)$$

and he lets

$$d = a + b + c$$

It is difficult to see why this additional rotation is required. I have suggested to Kennel that he delete it.

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Attachments
Figure 1
Table I

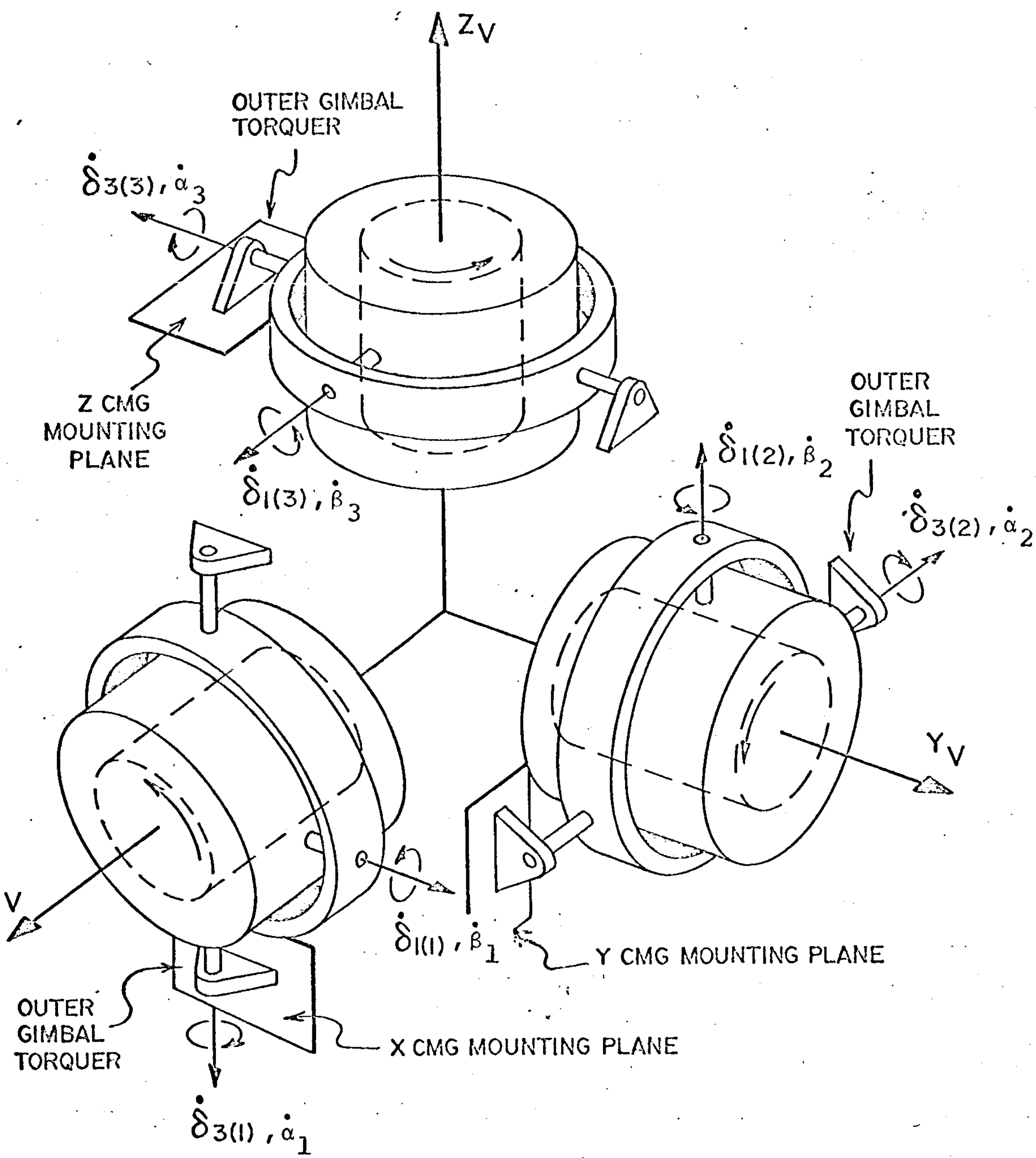

FIGURE 1. CONTROL MOMENT GYRO ORIENTATIONS

TABLE I

NOTATIONAL EQUIVALENCE

THIS MEMORANDUM	IPRD
$\underline{h}_i$	e_i
β_i	$\delta_{1(i)}$
α_i	$\delta_{3(i)}$
a	ε_{RA}
b	ε_{RB}
c	ε_{RC}
d	ε_{RT}
$\underline{\gamma}_i$	no equivalent
R_i	no equivalent
$\dot{\underline{\gamma}}_i = R_i\ \underline{\omega}_i$; $i = 1,2,3$	Eqs. 9.3.3.16-21
$R_1\ \underline{h}_2$	$\begin{pmatrix} S_{A11} \\ S_{A31} \end{pmatrix}$
$R_2\ \underline{h}_1$	$\begin{pmatrix} -S_{A12} \\ S_{A32} \end{pmatrix}$
$R_2\ \underline{h}_3$	$\begin{pmatrix} S_{B12} \\ S_{B32} \end{pmatrix}$
$R_3\ \underline{h}_2$	$\begin{pmatrix} -S_{B13} \\ S_{B33} \end{pmatrix}$
$R_3\ \underline{h}_1$	$\begin{pmatrix} S_{C13} \\ S_{C33} \end{pmatrix}$
$R_1\ \underline{h}_3$	$\begin{pmatrix} -S_{C11} \\ S_{C31} \end{pmatrix}$

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Subject: Analytical Development of the Skylab CMG Rotation Law

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